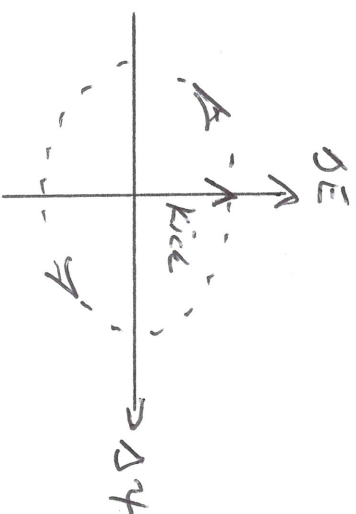


Accelerator Physics Lecture Modules

- 1. Introduction/Timing**
- 2. Beam propagation basics**
- 3. Simple Harmonic Oscillator**
- 4. Betatron oscillations (transverse motion)**
- 5. Dispersion**
- 6. Closed Orbit Distortion**
- 7. Synchrotron oscillations**
- 8. Properties of synchrotron radiation**
- 9. Radiation damping**
- 10. Equilibrium emittance**

Damped Single-Particle Motion

- o Photon emission results in loss of momentum
- o Momentum loss induces synchrotron oscillations
 - Change of circumference (momentum)
 - $(\Delta E, \Delta \gamma)$ oscillation

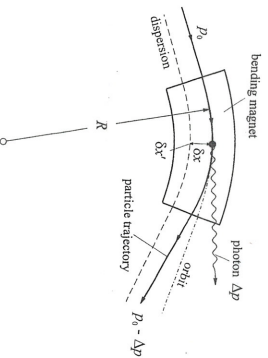


o Momentum loss induces betatron oscillations

- Change of closed orbit / dispersion
- (x, x') oscillation

$$x = \sqrt{\epsilon_x} \cos(\)$$

$$x' = \sqrt{\frac{\epsilon_x}{\rho}} \sin(\)$$



Synchrotron Oscillation Damping

$$E'' + 2\alpha_S E' + \pi_S^2 E = 0$$

$$\frac{1}{2T_0} \frac{du}{dE}$$

↑
oscillation
frequency

If $\frac{du}{dE} > 0$ motion is damped

$$\text{i.e. } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

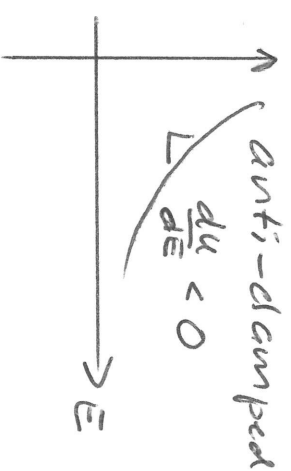
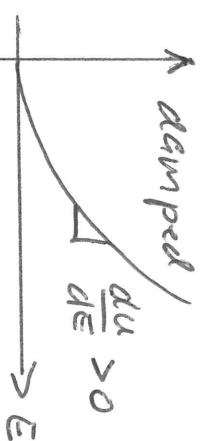
If $\frac{du}{dE} < 0$ motion is anti-damped

$$u = \oint P \cdot \frac{ds}{c} \quad \text{where } P \propto E^2 R^2$$

↑
Energy = Watts time

(loss/turn)

$u(E)$



Synchrotron Oscillation Damping

$$E'' + 2\alpha_S E' + \mathcal{R}_S^2 E = 0$$

$$\alpha_S = \frac{1}{T_0} \frac{du}{dE}$$

$$u = \int \mathcal{R}_S \frac{dS'}{c}$$

$$\text{where } dS' = \left(1 + \frac{\Delta x}{R}\right) ds$$

Substitute

$$\frac{du}{dE} = \int \left[\frac{d\mathcal{R}_S}{dE} + \frac{d}{dE} \left(\frac{\mathcal{R}_S}{R} \cdot D \frac{\partial P}{\partial P} \right) \right] ds$$

'1' term $\uparrow$ $\frac{\Delta x}{R}$ term

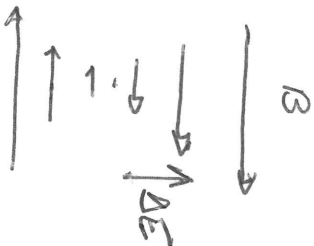
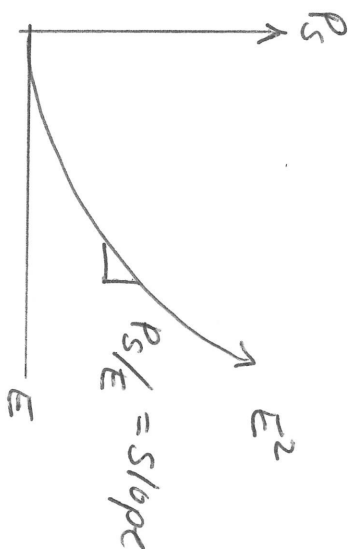
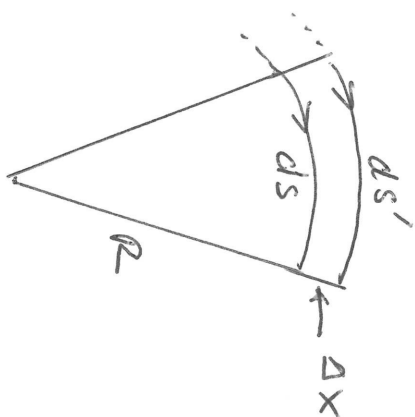
$$\text{Note } \mathcal{R}_S = C \cdot E^2 B^2$$

$$\frac{d\mathcal{R}_S}{dE} = C \cdot \left[2EB^2 + 2E^2 B \frac{dB}{dE} \right]$$

$$\frac{dP}{dE} = 2 \cdot \frac{\mathcal{R}_S}{E} + 2 \cdot \frac{\mathcal{R}_S}{B} \cdot \frac{dB}{dE}$$

$$\mathcal{R}_S \propto E^2$$

$\uparrow$ field change with energy
- dispersion in quadrupoles



Synchrotron Oscillation Damping

$$\alpha_S = \frac{1}{T_0} \frac{dU}{dE}$$

$$\frac{dU}{dE} = \frac{d}{dE} \left(\oint P_S ds' \right) \star$$

$$\left\{ \begin{array}{l} P_S \propto E^2 B^2 \\ \frac{dB}{dE} \text{ from dispersion} \end{array} \right.$$

Script $\mathcal{D}$ for dispersion effect

$$\alpha_S = \frac{U_0}{2 T_0 E} (2 + \mathcal{D})$$

$$\alpha_S = \frac{U_0}{T_0 E} + (\text{correction term})$$

$$(\mathcal{D}) = \frac{1}{c U_0} \oint P_S \left(\frac{20}{B} \frac{dB}{dx} + \frac{D}{R} \right) ds$$

$\uparrow$ quadrupoles
 $\uparrow$ dipoles

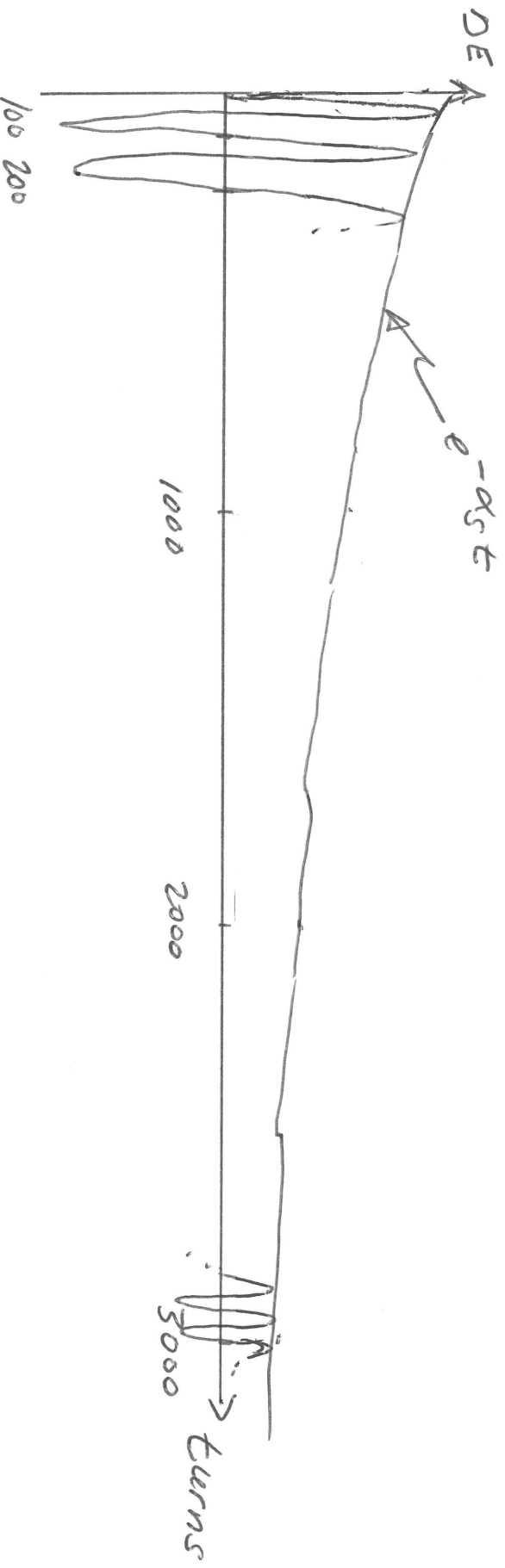
$\rightarrow$ need $\mathcal{D} > -2$

Synchrotron Oscillation Damping

$$\ddot{\epsilon} + 2\alpha_S \dot{\epsilon} + \omega_S^2 \epsilon = 0$$

$$\left\{ \begin{array}{l} \alpha_S = \frac{\omega_b}{2\omega_S} \left(2 + D \right) \quad e^{-\alpha_S t} \\ t_S = \frac{1}{\alpha_S} \sim \frac{\omega_b}{\omega_S} \sim \frac{10^{-6} \cdot 3 \times 10^9}{10^6} = 3 \times 10^{-3} \text{ s} = 3 \text{ ms} \quad (3000 \text{ turns}) \end{array} \right.$$

$$\left\{ \begin{array}{l} \omega_S \sim 10 \text{ kHz} \quad (100 \text{ turns}) \end{array} \right.$$

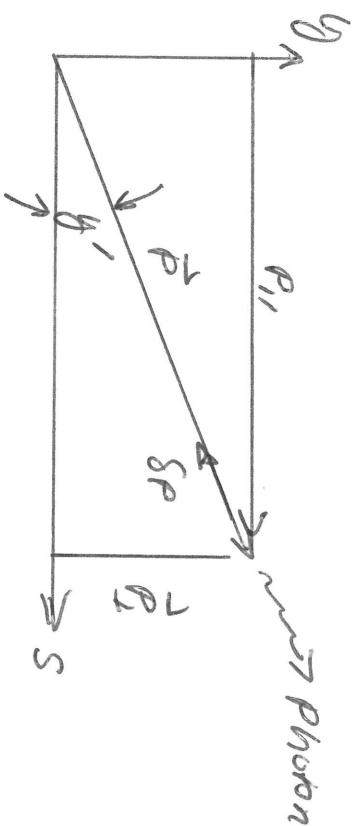


Vertical Betatron Oscillation Damping

o Take a different approach $\rightarrow$ shrinking phase ellipse

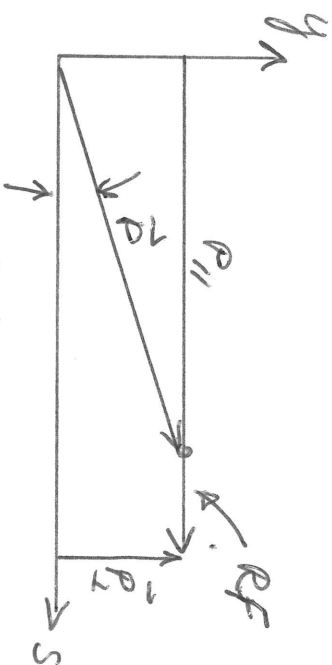
o How do vertical betatron oscillations get excited?

hints: $D_y \approx 0$, $\frac{d^2 p}{ds^2}$, Δs , Magnet alignment



emission event

$$\delta p \sim \frac{3 \text{ KeV}}{3 \text{ GeV}} = 10^{-6}$$

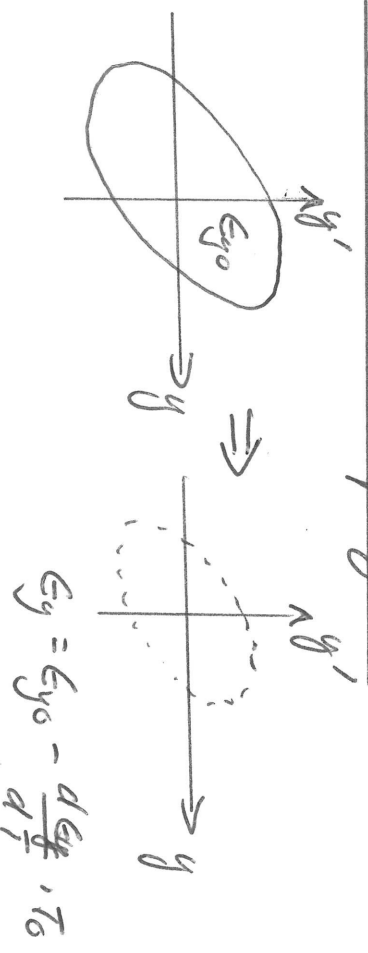


RF re-acceleration

first order $\delta y' = y' \cdot \left(\frac{\delta p}{p} \right)$

Vertical Betatron Oscillation Damping

Phase-Space Approach



$$\epsilon_y = \gamma y^2 + 2\alpha y y' + \beta y'^2$$

$$d(\epsilon_y) = \gamma(2y dy) + 2\alpha(y dy' + y' dy) + \beta(2y' dy')$$

$\swarrow$ damping term $\searrow$

$$dy' = -y' \frac{dp}{p}$$

y and y' are free betatron oscillations

$$y = \sqrt{\epsilon_{\beta}} \cos(\phi) \rightarrow \langle y \rangle = 0 \quad \langle y^2 \rangle = \frac{1}{2} \epsilon_{\beta y}$$

$$y' = \sqrt{\epsilon_{\gamma}} \sin(\phi) \rightarrow \langle y' \rangle = 0 \quad \langle y'^2 \rangle = \frac{1}{2} \epsilon_{\alpha}$$

$$\langle y y' \rangle = \frac{1}{2} \epsilon_{\gamma}$$

→ Substitute mean-square values into $d(\epsilon_y)$

Vertical Betatron Oscillation Damping

$$\epsilon_y = \delta y^2 + 2\alpha y y' + \beta y'^2$$

$$d(\epsilon_y) = 2\delta y y' + 2\alpha y dy' + 2\beta y' dy'$$

$$\langle y^2 \rangle = \frac{1}{2} \epsilon \beta$$

Twiss parameters

$$\langle y y' \rangle = \frac{1}{2} \epsilon \alpha$$

= moments of the distribution $f(x, x')$

$$\langle y'^2 \rangle = \frac{1}{2} \epsilon \gamma$$

Substitute, turn the crank ($\alpha^2 - \beta\gamma = 1$)

$$d(\epsilon_y) = -\epsilon_{y0} \left(\frac{\delta E}{E} \right)$$

minus

$$\frac{d\epsilon_y}{dt} = -\frac{1}{T_0} \epsilon_{y0} \left(\frac{u_0}{E} \right)$$

$\delta E / \text{turn}$

$$\int \rightarrow \epsilon_y(t) = \epsilon_{y0} e^{-\left(\frac{u_0}{T_0 E}\right) t}$$

attenuance

$$y(t) = \sqrt{\epsilon_y(t)} = \sqrt{\epsilon_{y0}} e^{-\left(\frac{1}{2} \frac{u_0}{T_0 E}\right) \cdot t}$$

from square root

$$\alpha_V = \frac{u_0}{2 T_0 E}$$

X-Y-Z Damping Rates

$$\alpha_S = \frac{1}{2T_0} \frac{u_0}{E} (2 + D) = (2 + D) \cdot \left(\frac{u_0}{2T_0 E} \right)$$

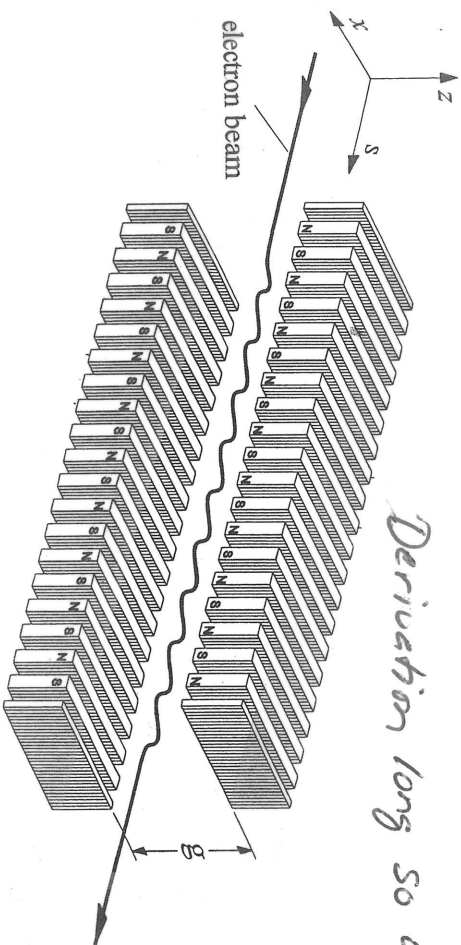
$$\alpha_y = \frac{1}{2T_0} \frac{u_0}{E} = (1) \cdot \left(\frac{u_0}{2T_0 E} \right)$$

Robinson (1958) → smart guy (legendary)

$$\alpha_S + \alpha_y + \alpha_x = 4 \left(\frac{u_0}{2T_0 E} \right)$$

$$\alpha_x = (1 - D) \left(\frac{u_0}{2T_0 E} \right)$$

Derivation long so we skip it...



Robinson damping wigglers

More on Robinson Criteria

$$\alpha_{\text{total}} = \alpha_x + \alpha_y + \alpha_s = 4 \left(\frac{W_0}{T_0 E} \right) = 2 \left(\frac{W_0}{T_0 E} \right)$$

↑

Oscillation amplitude
damping

$$\frac{J_x + J_y + J_s}{\text{Partition numbers}} = \frac{2 \langle P \rangle}{E} = \frac{2}{T_{\text{Energy damping}}}$$

$$\frac{1}{\alpha_{\text{amp1}}} = T_{\text{amp1}} = \frac{T_{\text{Energy}}}{2}$$

$$E_{\text{Energy}}(t) = E_0 e^{-t/T_{\text{Energy}}}$$

$$A_{\text{amp1}}(t) = \sqrt{E_0} e^{-t/(T_0/2)}$$

$T_{\text{amp1}} = T_{\text{Energy}}/2$

$P, W_0, D, J_{x,y,z}, E, G_E \rightarrow \text{Synchrotron Integrals}$

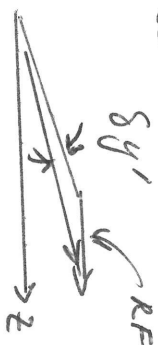
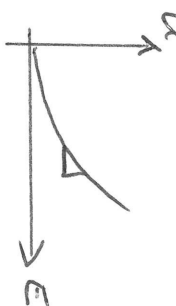
Review

- Photon emission produces synchrotron and betatron oscillations
— quantum excitation

- Oscillations are damped because

1) RF re-acceleration

2) $\frac{du}{dE} > 0$



high energy particles radiate more (friction)

- Oscillation damping rates $\sim \frac{T_0 E}{u_0} \sim \text{few ns}$

- Correction term D depends on Twiss parameters (α, β, γ)
Dispersion (D, D')

- Robinson criteria on 6-D phase space volume