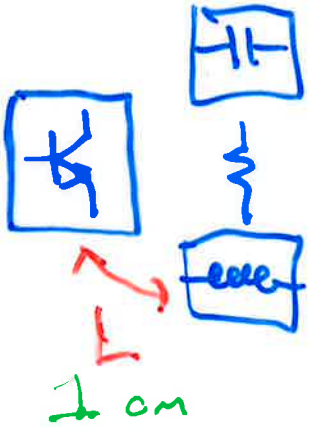
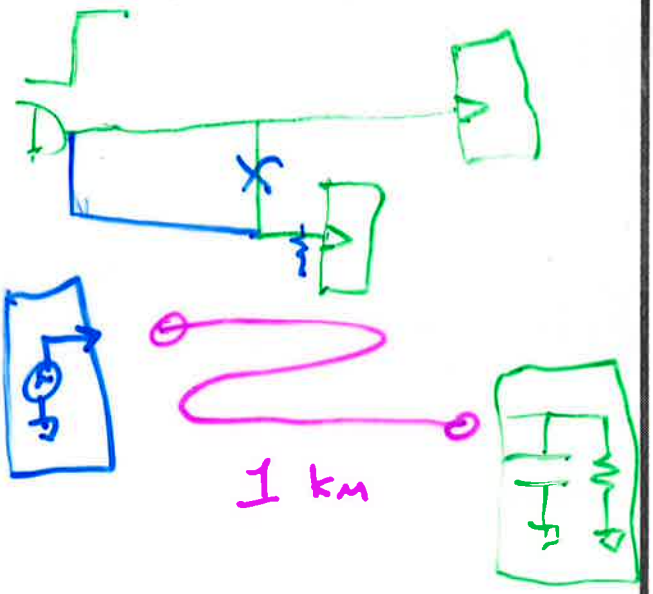
Circuit Dimensions, Signal Dimensions

When do lumped-element circuits start to become transmission line circuits?

Wavelength - 30 MHz = 10 mt

3 GHz = 10 cm

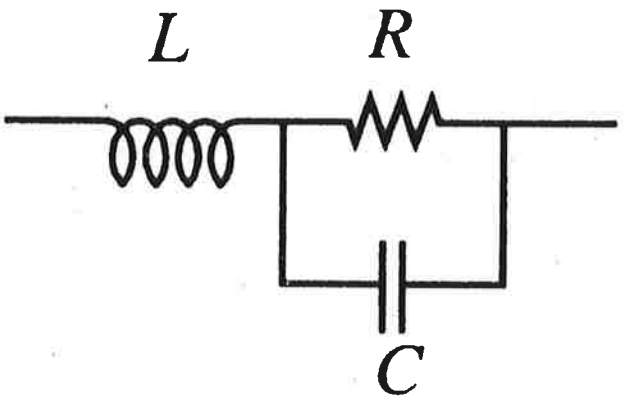




Lumped-element circuits

Transmission-line circuits

- coaxial
- stripline, microstrip, coplanar, etc

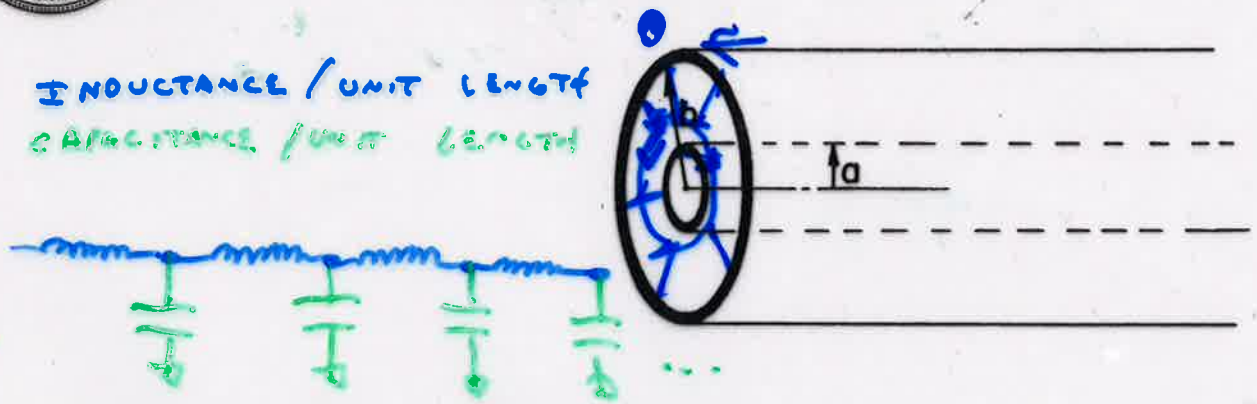


Simple lumped RF resistor model

Transmission lines

INDUCTANCE / UNIT LENGTH

CAPACITANCE / UNIT LENGTH



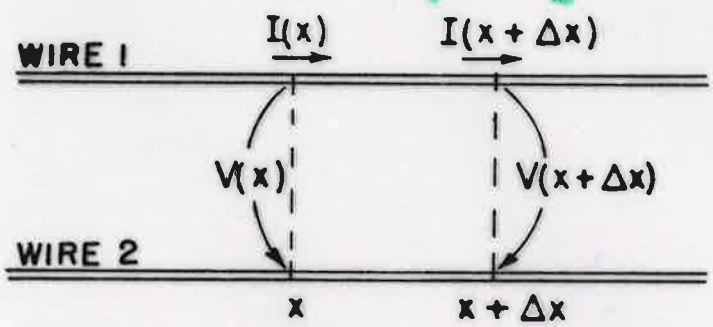


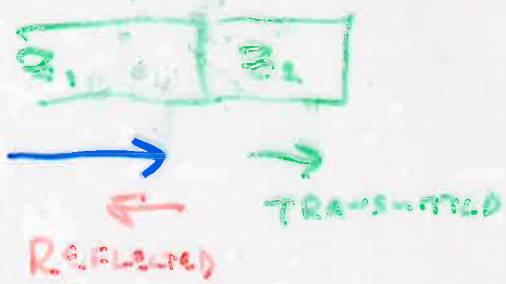
Fig. 24–2. The currents and voltages of a transmission line.

IMPEDANCE $Z = \frac{V}{I}$

$Z \propto \sqrt{\frac{L}{C}}$

$V_{prop} \propto \sqrt{\frac{1}{LC}}$

Reflection, transmission at a discontinuity



$$R = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

Transmission Lines

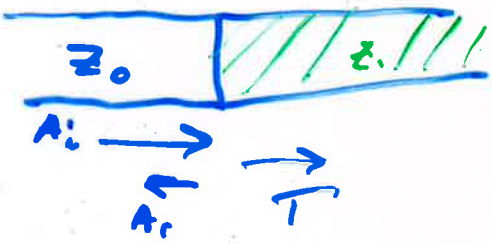
Characteristic impedance $Z = K_1 \sqrt{\frac{L}{C}}$

inductance/unit length L

capacitance/unit length C

group velocity $V = K_2 \sqrt{\frac{1}{LC}}$

Reflections at a discontinuity



$$R = \frac{Z_1 - Z_0}{Z_0 + Z_1}$$

Matching networks - matching techniques

TRANSMISSION LINE TERMINATED IN Z_{CHAR} LOOKS PURELY REAL

1/4 WAVE MATCHING

$$Z_M = \sqrt{Z_0 Z_1}$$

3.3.1 Parallel Wires

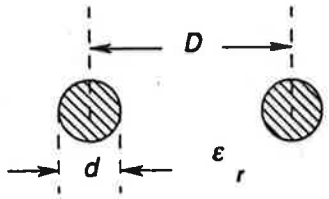


Figure 3.3.1.1: Parallel Wires

$$Z_0 = \frac{\eta_0}{\pi\sqrt{\varepsilon_r}} \cosh^{-1}\left(\frac{D}{d}\right) \ (\Omega)$$

or

$$Z_0 = \frac{\eta_0}{2.0\,\pi\sqrt{\varepsilon_r}} \cosh^{-1}\left(\frac{2.0\,D^2 - d^2}{d^2}\right) \ (\Omega)$$

valid for

$d/D \leq 1.0$

3.3.3 Twisted Pair

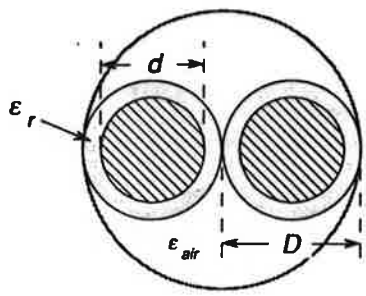


Figure 3.3.3.1: Twisted Pair

The twisted pair provides good low frequency shielding. Undesired signals tend to be coupled equally into each line of the pair. A differential receiver will therefore completely cancel the interference. Lefferson [3] gives design equations for this configuration:

$$Z_0 = \frac{\eta_0}{\pi\sqrt{\varepsilon_{eff}}} \cosh^{-1}\left(\frac{D}{d}\right) \ (\Omega) \qquad (3.3.3.1)$$

$$\varepsilon_{eff} = 1.0 + q\,(\varepsilon_r - 1.0) \qquad (3.3.3.2)$$

$$q = 0.25 + 0.0004\,\theta^2 \qquad (3.3.3.3)$$

$$T = \frac{\tan\theta}{\pi D} = \text{twists per length} \qquad (3.3.3.4)$$

3.2.1 Round Coaxial Cable

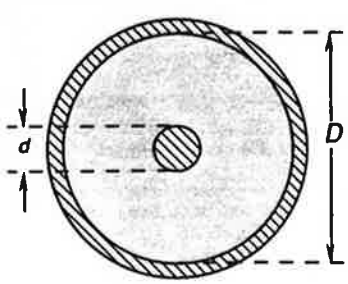


Figure 3.2.1.1: Round Coaxial Cable

The equations for this transmission line can be derived easily and are exact for the case of an infinitely long line (no fringing fields):

$$Z_0 = \frac{\eta_0}{2.0\,\pi\sqrt{\varepsilon_r}} \ln\left(\frac{D}{d}\right) \ (\Omega) \qquad (3.2.1.1)$$

$$\alpha_c = \left(\frac{0.014272\sqrt{f}}{Z_0}\right)\left(\frac{1}{d} + \frac{1}{D}\right) \ (\text{dB / m}) \qquad (3.2.1.2)$$

$$\alpha_d = 0.091207\, f\sqrt{\varepsilon_r}\tan\delta \ (\text{dB / m}) \qquad (3.2.1.3)$$

3.3.4 Five-Wire Line

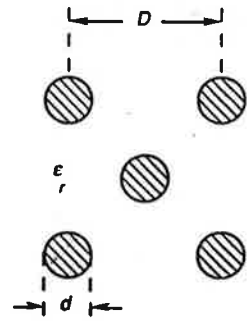


Figure 3.3.4.1: Five-Wire Line

$$Z_0 = \frac{2.5044\,\eta_0}{\pi\sqrt{\varepsilon_r}} \ln\left[\frac{D}{0.933\,d}\right] \ (\Omega)$$

3.3.2 Unequal Size Parallel Wires

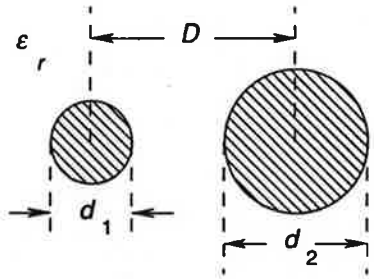


Figure 3.3.2.1: Unequal Size Parallel Wires

$$Z_0 = \frac{\eta_0}{2.0\,\pi\sqrt{\varepsilon_r}} \cosh^{-1}\left(\frac{4.0\,D^2 - d_1^2 - d_2^2}{2.0\,d_1\,d_2}\right) \ (\Omega) \qquad (3.3.2.1)$$

3.2.4 Square Coaxial Cable

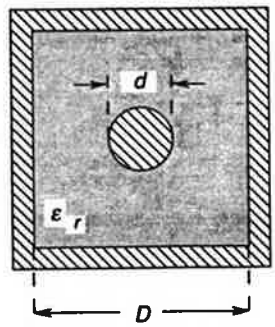


Figure 3.2.4.1: Square Coaxial With Circular Center Conductor

The square coaxial wire configurations are useful at very high frequencies as smaller replacements for waveguide. The square cross section makes them easier to fabricate than a round coax.

For the round center conductor as shown in Figure 3.2.4.1:

$$Z_0 = \frac{\eta_0}{2\,\pi\sqrt{\varepsilon_r}} \ln\left[\frac{1.0787\,D}{d}\right] \ (\Omega) \qquad (3.2.4.1)$$

and for $Z_0 \leq 2.0\ \Omega$

$$Z_0 = 21.2\,\sqrt{D/d - 1.0} \ (\Omega) \qquad (3.2.4.2)$$

3.2.5 Rectangular Coaxial Line

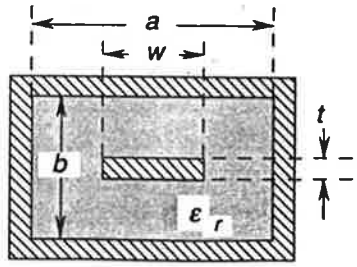


Figure 3.2.5.1: Rectangular Coaxial Line

This structure is a transitional structure between round coax and stripline, microstrip line or other planar lines. Many attempts have been made to utilize this structure for photolithographic construction of fully shielded controlled impedance lines. Rotating the center conductor relative to the shield also allows this structure to have a continuously variable characteristic impedance [4].

As reported in [9] rectangular coax can be used in PCBs constructed with additive techniques to obtain a completely shielded structure. The process by Augat Microtec build the structure using photolithographic techniques from polyimide and copper.

A related structure [14], Figure 3.2.5.2, was constructed with thick-film techniques. The main limitation appears to have been the thickness, b, achievable without excessive layered printings. Without a sufficiently thick dielectric, center conductor to shield shorts were common. Low dielectric constant pastes were recommended.

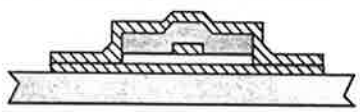


Figure 3.2.5.2: Thick-Film Coaxial Line

3.2.6 Trough Line or Channel Line

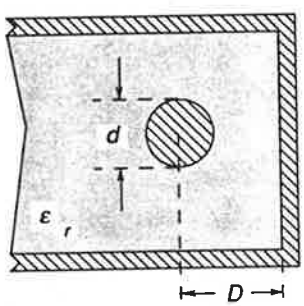


Figure 3.2.6: Trough Line

Wheeler [2] gives:

$$Z_0 = \frac{\eta_0}{4\pi\sqrt{\varepsilon_r}} \ln\left[1.0 + \left\{\frac{1}{2}\left(\frac{4}{\pi}\tanh\frac{\pi}{2}\right)^2\left[(D/d)^2 - 1.0\right]\right\} + \sqrt{\left\{\frac{1}{2}\left(\frac{4}{\pi}\tanh\frac{\pi}{2}\right)^2\left[(D/d)^2 - 1.0\right]\right\}^2 + \frac{4}{9}\left[(D/d)^2 - 1.0\right]}\right] \quad (\Omega) \qquad (3.2.6)$$

with a stated accuracy of about 1%.

3.2.7 Strip-Centered Coaxial Line

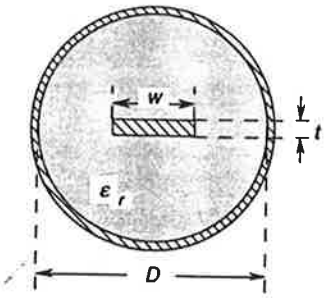


Figure 3.2.7.1: Strip-Centered Coaxial Cable

The configuration above is used to obtain lower loss by building a coaxial cable with a larger center conductor surface area. Bongianni [1] reports cables made with extremely fine dimensions—the center conductor was 150 µm × 12.5 µm. For $Z_0 \le 30.0\,\pi/\sqrt{\varepsilon_r}$ (Ω):

$$Z_0 = \frac{15.0\,\pi^2}{\sqrt{\varepsilon_r}} \frac{1.0}{\ln\left[\frac{2.0(D+w)}{D-w}\right]} \quad (\Omega) \qquad (3.2.7.1)$$

For $Z_0 \ge 30.0\,\pi/\sqrt{\varepsilon_r}$ (Ω):

$$Z_0 = \frac{60.0}{\sqrt{\varepsilon_r}} \ln\left(\frac{2.0\,D}{w}\right) \quad (\Omega) \qquad (3.2.7.2)$$

There is a bit of a chicken-and-the-egg problem here because it is necessary to know Z_0 in order to choose the equation for Z_0; however, the two equations pass quite close and it is okay to use either to make the choice.

3.2.3 Eccentric Round Coaxial Cable

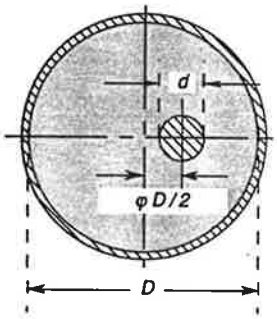
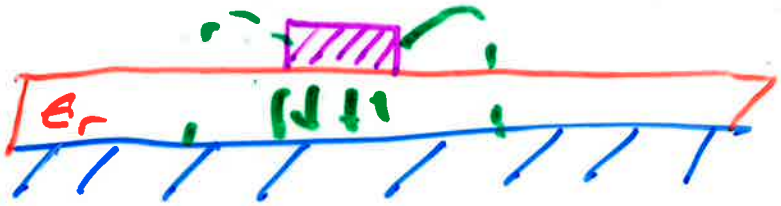


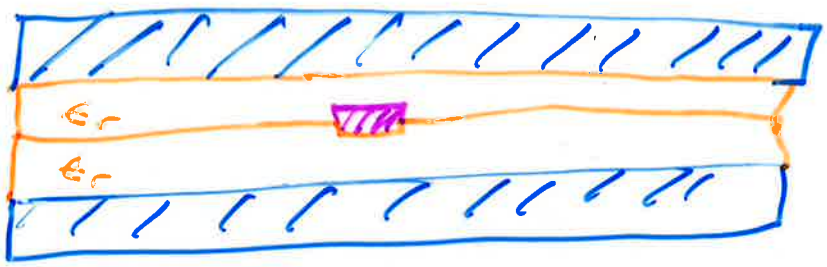
Figure 3.2.3.1: Eccentric Round Coaxial Cable

The eccentric coax equations enable us to analyze the effects of tolerances in the manufacture of the cable. This structure has also been used as a continuously adjustable $\lambda/4$ line. The center conductor is moved within the outer shield by a mechanical probe resulting in smooth variation of the characteristic impedance.

For center conductors off center in one direction, [1] gives:

$$Z_0 = \frac{\eta_0}{2.0\,\pi\sqrt{\varepsilon_r}} \cosh^{-1}\left[\frac{D}{2.0\,d}\left(1.0 - \varphi^2\right) + \frac{d}{2.0\,D}\right] (\Omega) \qquad (3.2.3.1)$$





3.3.5 Paired Strips

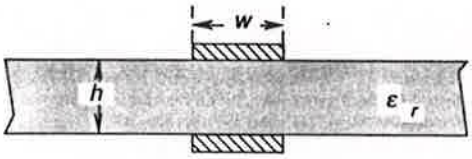


Figure 3.3.5.1: Paired Strips

For wide strips ($a / b > 1$):

$$Z_0 = \frac{\eta_0}{\sqrt{\varepsilon_r}} \left\{ \frac{a}{b} + \frac{1.0}{\pi} \ln 4 + \frac{\varepsilon_r + 1.0}{2\,\pi\,\varepsilon_r} \ln\left[\frac{\pi\,\varepsilon\,(a / b + 0.94)}{2.0} \right] + \frac{\varepsilon_r - 1.0}{2\,\pi\,\varepsilon_r^2} \ln \frac{\varepsilon\,\pi^2}{16.0} \right\}^{-1} \ (\Omega) \qquad (3.3.5.1)$$

Stated error is less than 1% for wide strips.

For narrow strips ($a / b < 1$):

$$Z_0 = \frac{\eta_0}{\pi\sqrt{\varepsilon_r}} \left[\ln \frac{4.0\,b}{a} + \frac{1.0}{8.0} \left(\frac{a}{b}\right)^2 - \frac{\varepsilon_r - 1.0}{2.0\,(\varepsilon_r + 1.0)} \left(\ln \frac{\pi}{2.0} + \frac{\ln \frac{4.0}{\pi}}{\varepsilon_r} \right) \right] \ (\Omega) \qquad (3.3.5.2)$$

where

$$a = w / 2.0 \qquad (3.3.5.3)$$

3.4.1 Coplanar Waveguide

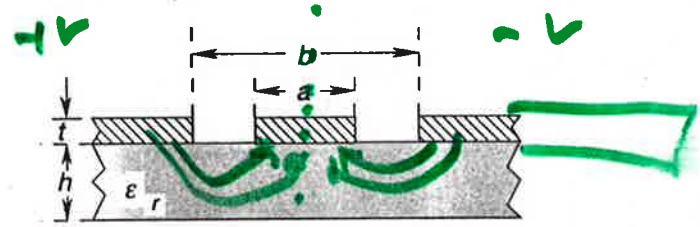


Figure 3.4.1.1: Coplanar Waveguide

The coplanar waveguide (CPW) configuration's advantages stem from its single-sided nature. Grounding components does not require plated through-holes to a plane on the other side of the substrate. This makes it ideal for use with surface mounted components. Another feature of coplanar waveguide is that we can narrow traces to match component lead widths while keeping Z_0 constant. The ground plane should extend greater than $5b$ on each side of the gap or the coplanar strip analysis should be used. The ground planes on either side of the center conductor may need to be connected periodically with wire jumpers depending on the frequency. The enclosure's cover can be used to jumper the two ground planes if it is kept away by at least $(b - a)$. To prevent propagation of higher modes, b should be less than $\lambda / 2$.

The design equations for coplanar waveguide are:

$$Z_0 = \frac{30.0\,\pi}{\sqrt{\varepsilon_{eff,1}}} \frac{K(k_1')}{K(k_1)} \qquad (3.4.1.1)$$

$$\varepsilon_{eff,1} = \varepsilon_{eff} - \frac{\varepsilon_{eff} - 1.0}{\frac{(b - a) / 2.0}{0.7\,t} \frac{K(k)}{K'(k)} + 1.0} \qquad (3.4.1.2)$$

$$\varepsilon_{eff} = 1.0 + \frac{\varepsilon_r - 1.0}{2.0} \frac{K(k')K(k_1)}{K(k)K(k_1')} \qquad (3.4.1.3)$$

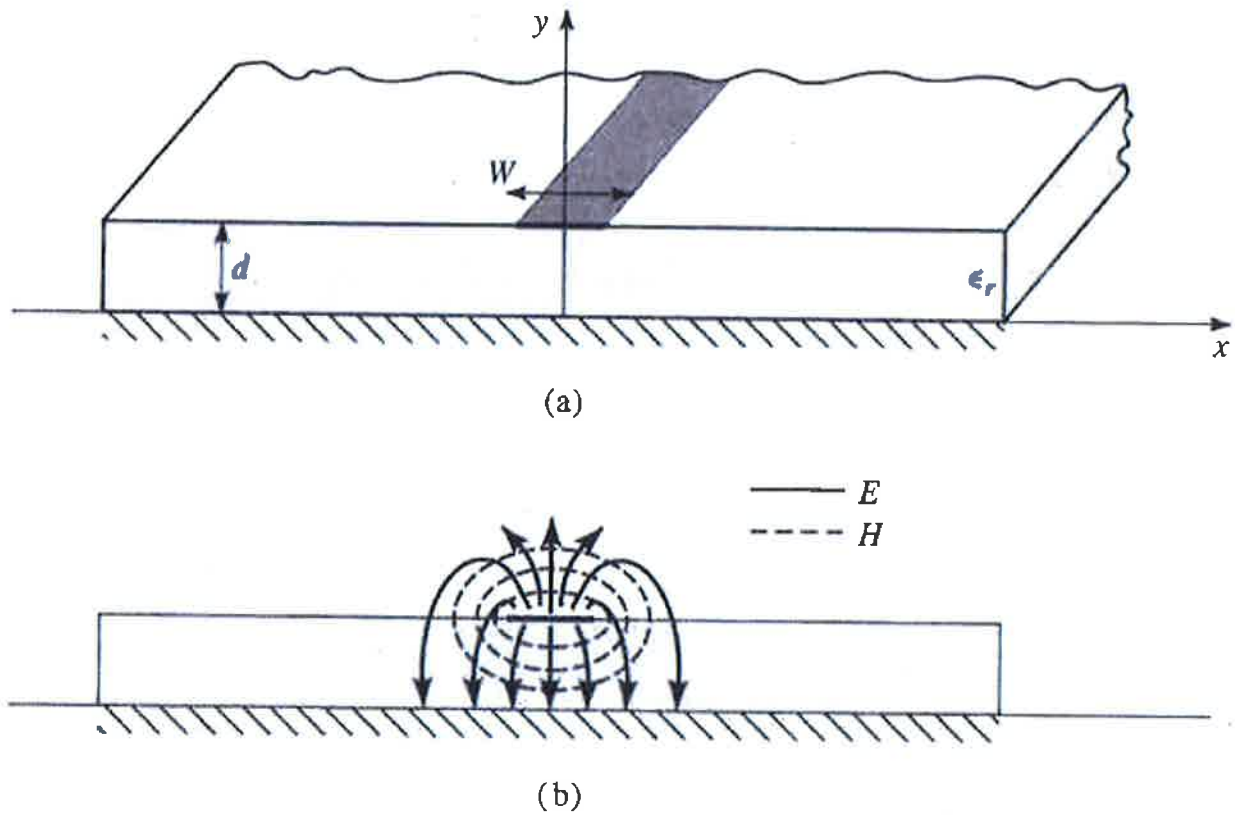
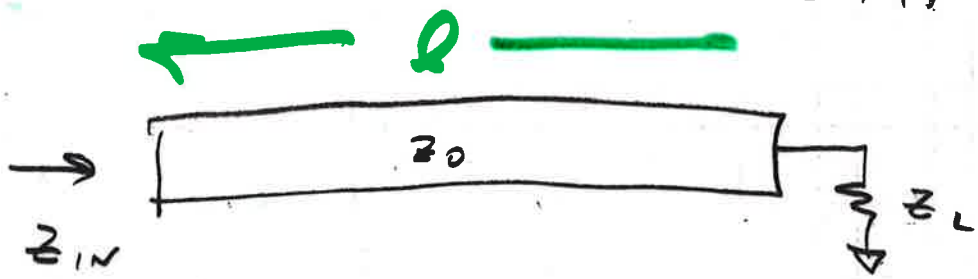


FIGURE 3.25 Microstrip transmission line. (a) Geometry. (b) Electric and magnetic field lines.

IMPEDANCE OF ARBITRARY TERMINATION

LOSSLESS LINE (FROM POZAR, 2.44)



Phase velocity $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

WAVELENGTH $\lambda = \frac{2\pi}{\beta}$

$$Z_{in} = Z_0 \frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l}$$

REFLECTION $\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$

SPECIAL CASES TO CONSIDER

$Z_L = \infty$ (OPEN) $\qquad Z_L = 0$ (SHORT)

OPEN CIRCUITED LINE

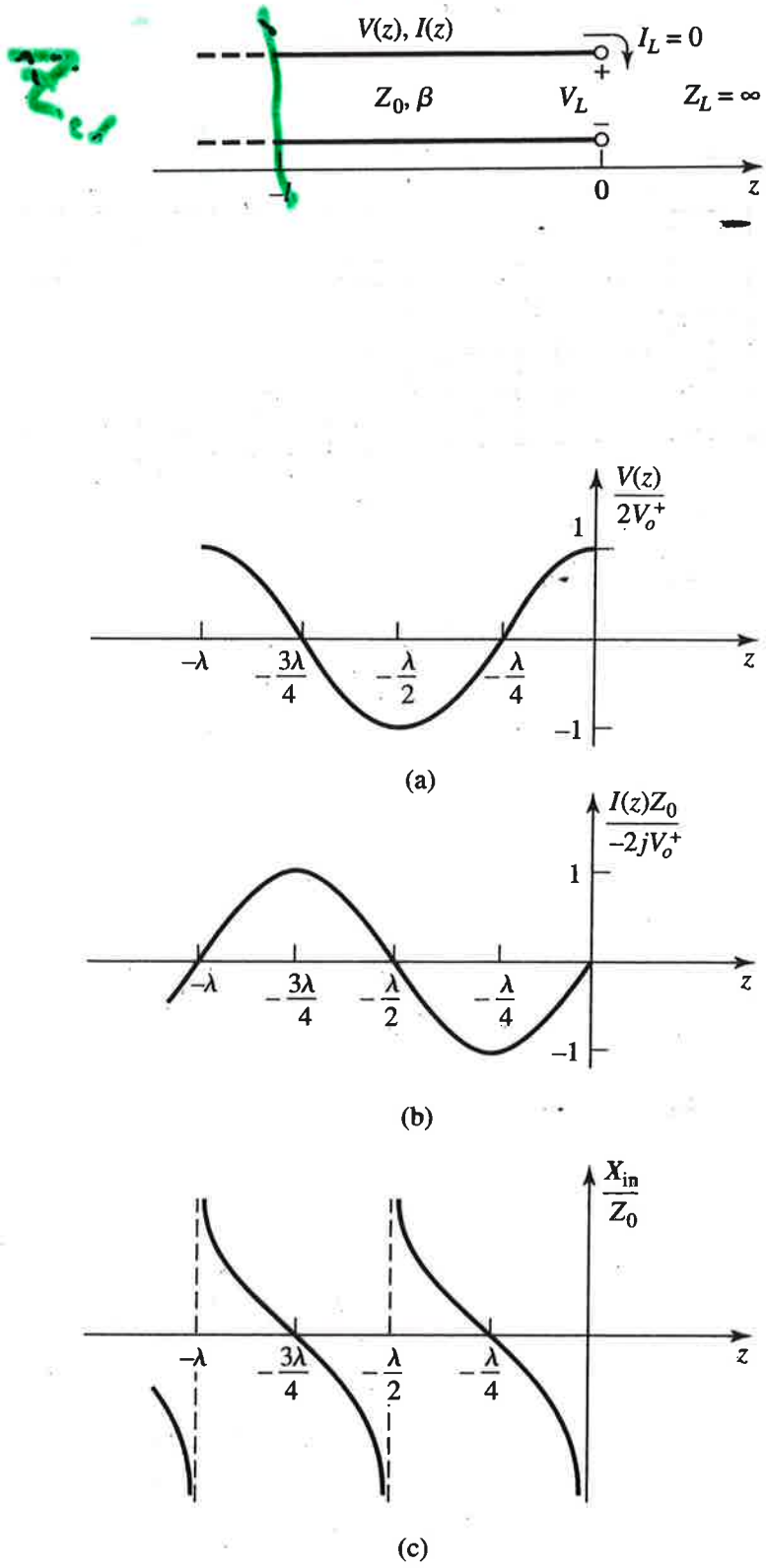


FIGURE 2.8 (a) Voltage, (b) current, and (c) impedance ($R_{in} = 0$ or ∞) variation along an open-circuited transmission line.

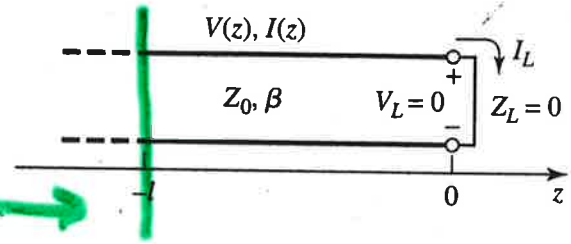


FIGURE 2.5 A transmission line terminated in a short circuit.

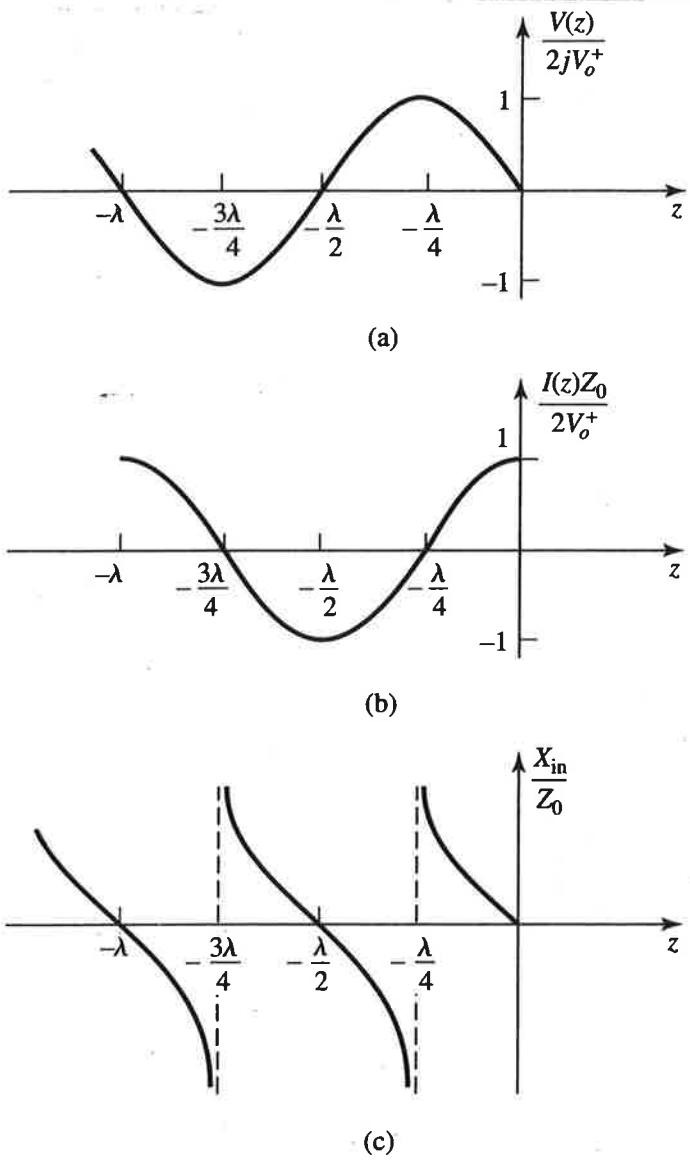
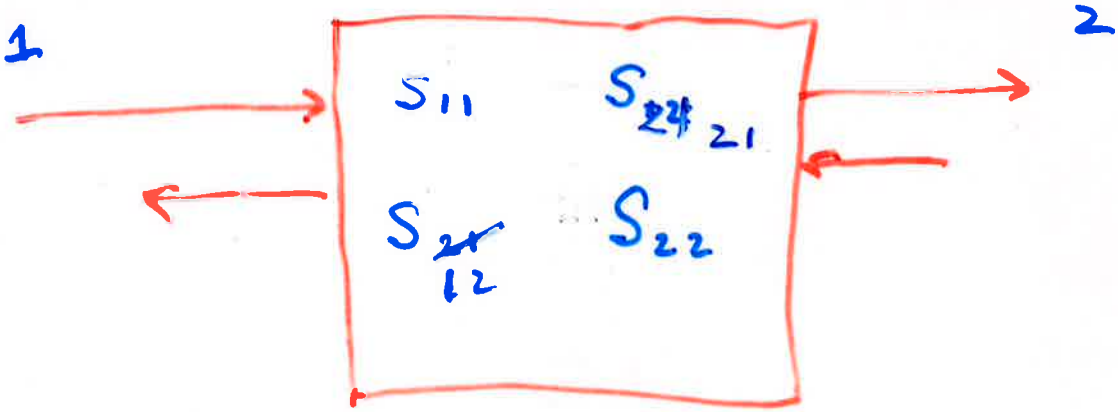


FIGURE 2.6 (a) Voltage, (b) current, and (c) impedance ($R_{in} = 0$ or ∞) variation along a short-circuited transmission line.

S parameters

Characterize system via scattering matrix S (COMPLEX)

TWO-PORT CONCEPT



S_{11} (REFLECTION BACK TOWARDS PORT 1)

S_{21} (FORWARD GAIN 1 → 2)

S_{22} (REFLECTION BACK TOWARDS 2, from 2 port)

S_{12} (REVERSE GAIN 2 → 1)

Important - definition of reference plane

Network Analyzer makes these measurements

Network Analyzer is worthless without good calibration standards

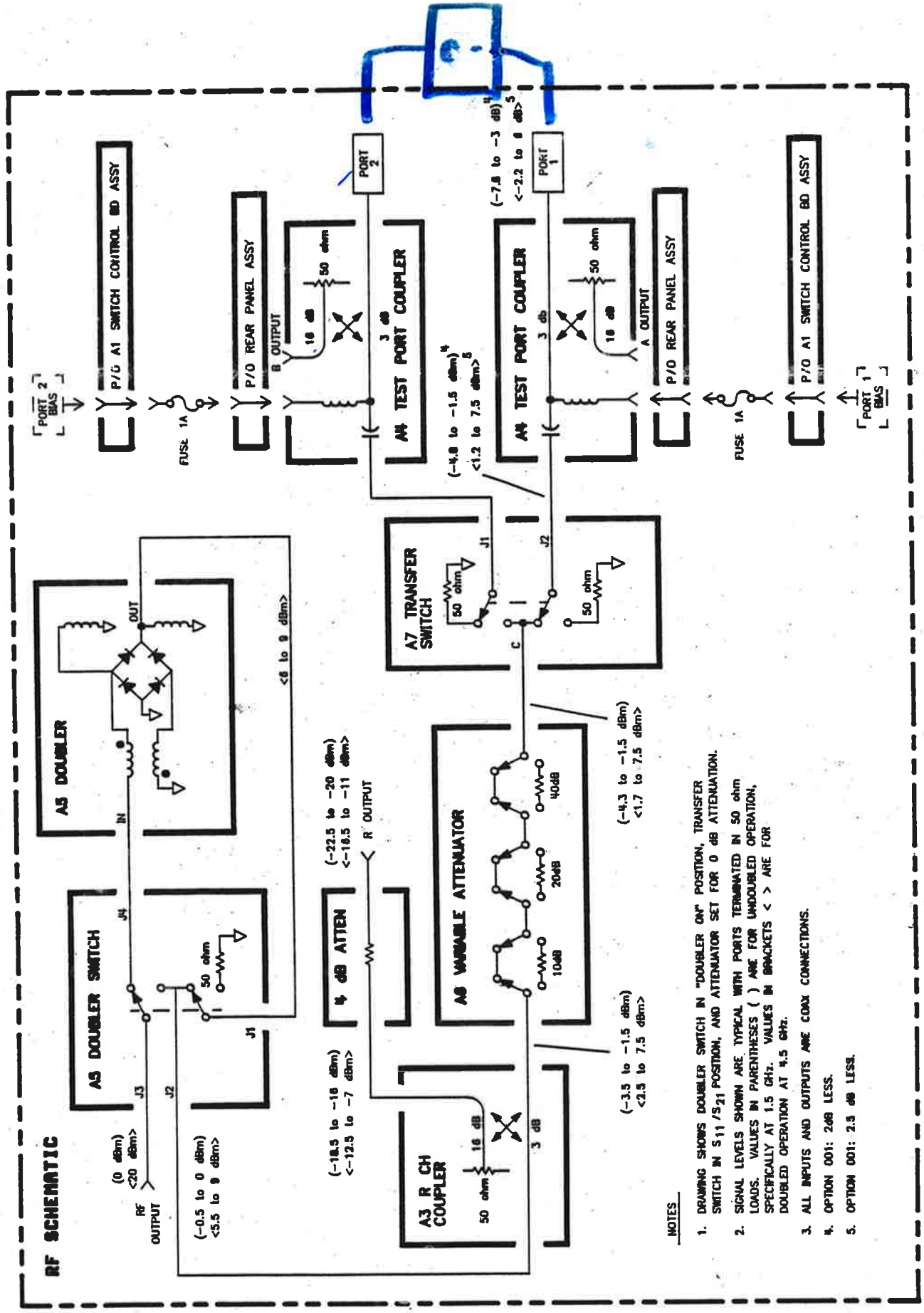


Figure 18. RF Power Levels at (1.5 GHz) and <4.5 GHz>

A network analyzer system consists of a source, signal separation devices (a test set or power splitter), a receiver, and a display. The HP 8753A RF vector network analyzer integrates a high resolution synthesized RF source and a dual channel three-input receiver for measurement and display of test device characteristics. Figure 1 is a simplified block diagram of the HP 8753A network analyzer system.

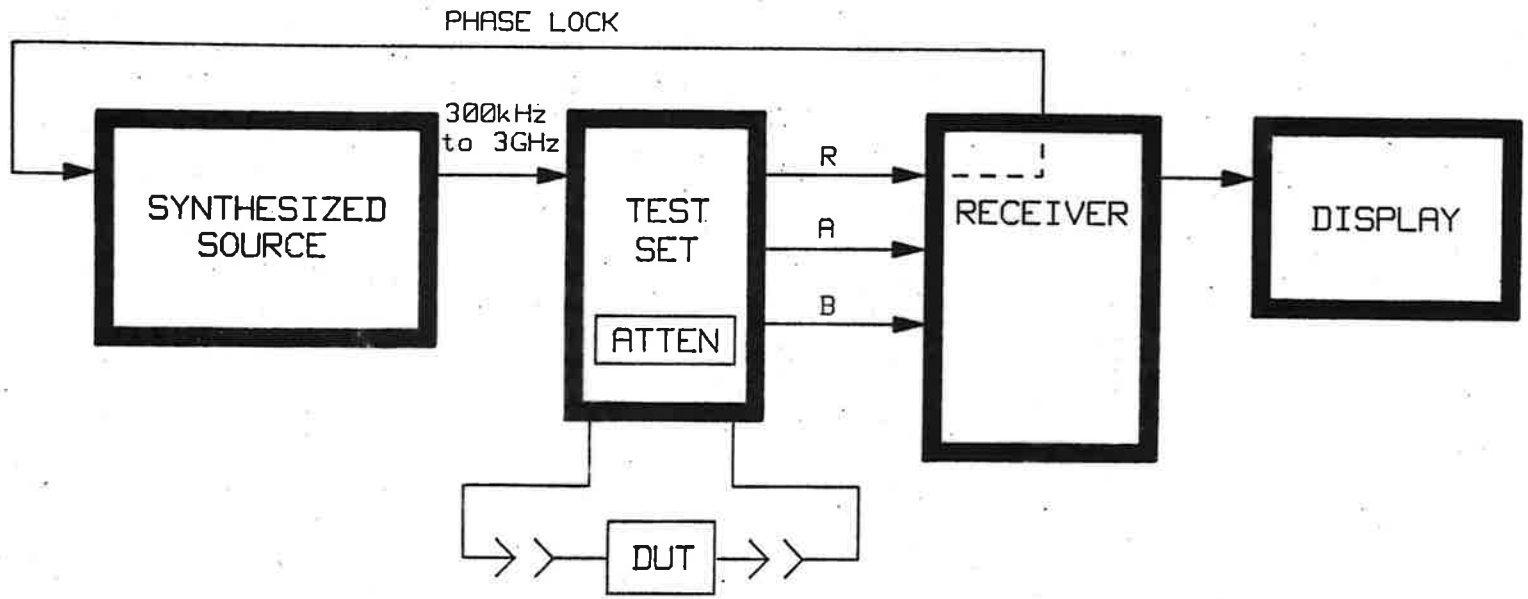


Figure 1. Simplified System Block Diagram

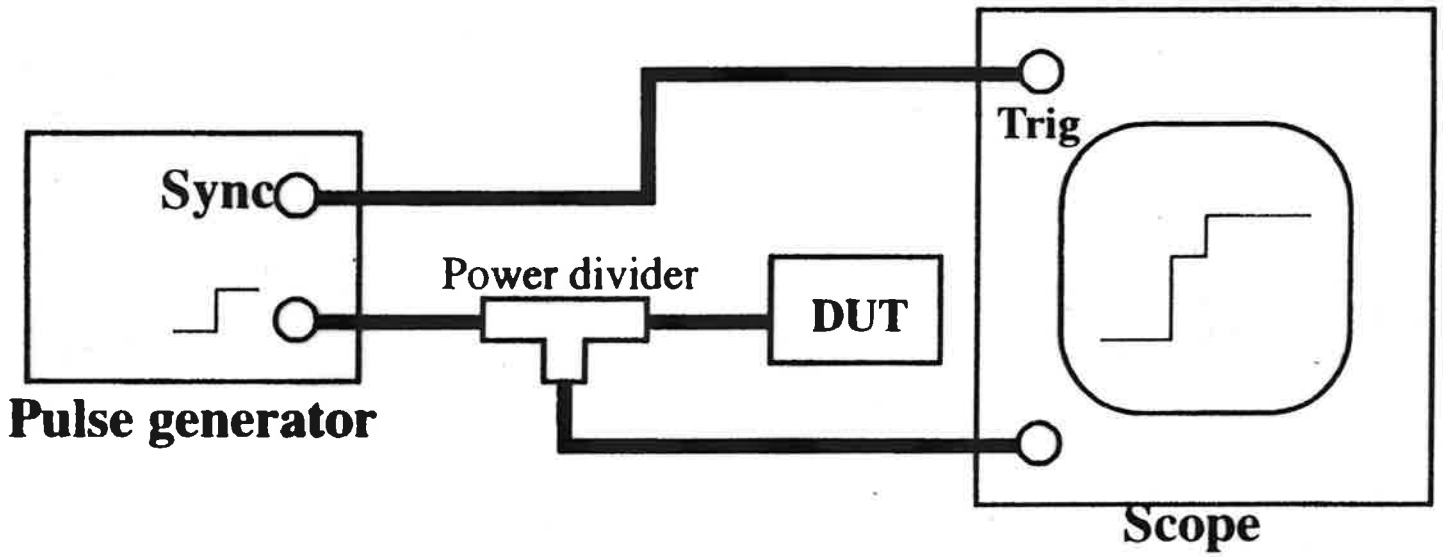


FIGURE 8.1. Time-domain reflectometer

A time domain reflectometer setup is shown in Figure 3.

The step generator produces a positive-going incident wave that is applied to the transmission system under test. The step travels down the transmission line at the velocity of propagation of the line. If the load impedance is equal to the characteristic impedance of the line, no wave is reflected and all that will be seen on the oscilloscope is the incident voltage step recorded as the wave passes the point on the line monitored by the oscilloscope. Refer to Figure 4.

If a mismatch exists at the load, part of the incident wave is reflected. The reflected voltage wave will appear on the oscilloscope display algebraically added to the incident wave. Refer to Figure 5.

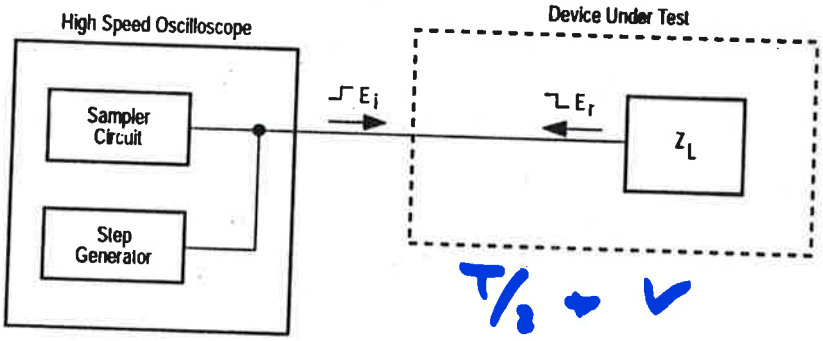


Figure 3. Functional block diagram for a time domain reflectometer

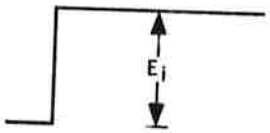


Figure 4. Oscilloscope display when $E_r = 0$

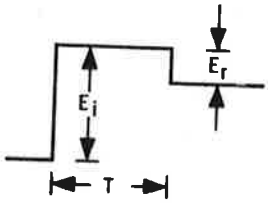


Figure 5. Oscilloscope display when $E_r \neq 0$

Analyzing Reflections

The shape of the reflected wave is also valuable since it reveals both the nature and magnitude of the mismatch. Figure 6 shows four typical oscilloscope displays and the load impedance responsible for each. Figures 7a and 7b show actual screen captures from the 86100B. These displays are easily interpreted by recalling:

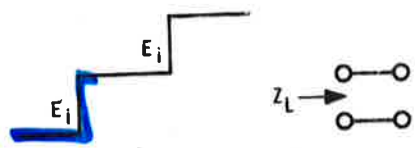
$$\rho = \frac{E_r}{E_i} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

Knowledge of E_i and E_r, as measured on the oscilloscope, allows Z_L to be determined in terms of Z_o, or vice versa. In Figure 6, for example, we may verify that the reflections are actually from the terminations specified.

eflected wave is also valuable since it reveals both
gnitude of the mismatch. Figure 6 shows four typical
ays and the load impedance responsible for each.
show actual screen captures from the 86100B.
easily interpreted by recalling:

$$\rho = \frac{E_r}{E_i} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

nd E_r, as measured on the oscilloscope, allows Z_L
in terms of Z_o, or vice versa. In Figure 6, for
verify that the reflections are actually from the
ified.



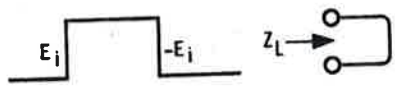
(A) Open Circuit Termination ($Z_L = \infty$)

(A) $E_r = E_i$

Therefore $\frac{Z_L - Z_o}{Z_L + Z_o} = +1$

Which is true as $Z_L \rightarrow \infty$

$\therefore Z$ = Open Circuit



(B) Short Circuit Termination ($Z_L = 0$)

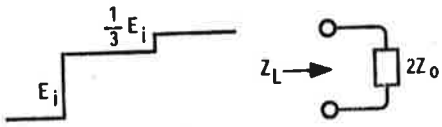
(B) $E_r = E_i$

Therefore $\frac{Z_L - Z_o}{Z_L + Z_o} = -1$

Which is only true for finite Z

When $Z_L = 0$

$\therefore$ Z = Short Circuit

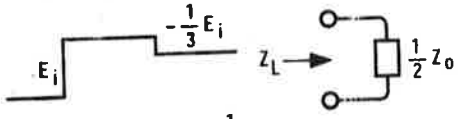


(C) Line Terminated in $Z_L = 2Z_o$

(C) $E_r = +\frac{1}{3}E_i$

Therefore $\frac{Z_L - Z_o}{Z_L + Z_o} = +\frac{1}{3}$

and $Z_L = 2Z_o$



(D) Line Terminated in $Z_L = \frac{1}{2}Z_o$

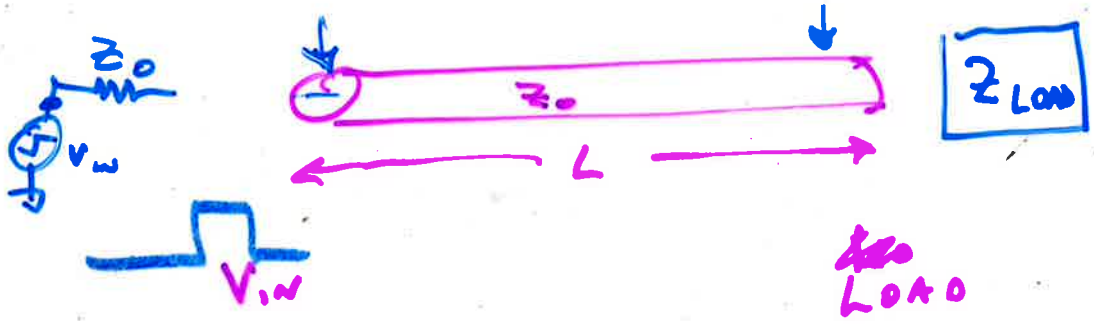
(D) $E_r = -\frac{1}{3}E_i$

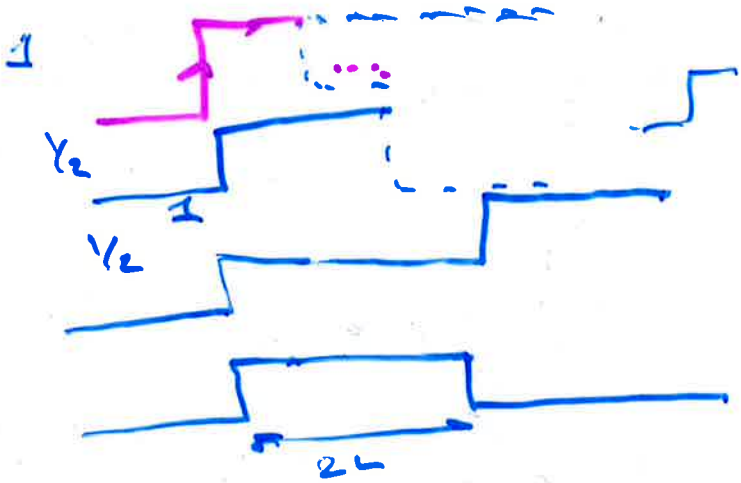
Therefore $\frac{Z_L - Z_o}{Z_L + Z_o} = -\frac{1}{3}$

and $Z_L = \frac{1}{2}Z_o$

Figure 6. TDR displays for typical loads.

SNAP QUIZ



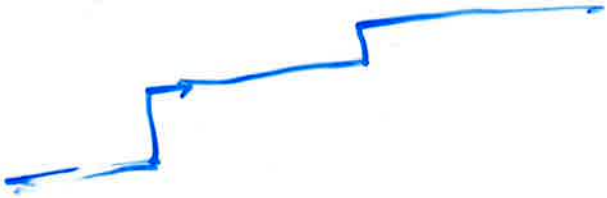


Z_0

OPEN

SHORT

$Z_L < Z_0$



$Z_L > Z_0$

SHORT

OPEN

The Complete Smith Chart

Black Magic Design